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COMPARISON OF TURBINE ANNULUS MASS FLOW COMPUTED BY ONE- AND TWO-DIMENSIONAL ANALYSIS

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16. Abstract

Variations in specific heat ratio, flow angle, critical velocity ratio, swirl distribution exponent, and radius ratio were considered in computing the mass flow. Variations in specific heat ratio had no significant effect and variations in critical velocity ratio had only small effect on computed mass flow between a one- and two-dimensional analysis. All non-free-vortex cases considered showed larger differences in computed mass flow between one- and two-dimensional analysis than for free vortex flow. For the non-free-vortex cases, decreasing radius ratio and increasing flow angle resulted in larger differences in mass flow as computed by the two methods.

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COMPARISON OF TURBINE ANNULUS MASS FLOW COMPUTED BY ONE- AND TWO-DIMENSIONAL ANALYSIS

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SUMMARY

An analytical investigation was conducted to determine the difference in computed mass flow rate between a one-dimensional and a two-dimensional analysis for a turbine annulus using various radial distributions of swirl velocity. Variables considered in the analysis included swirl velocity distribution, flow angle, critical velocity ratio, radius ratio, and specific heat ratio.

The results of the analysis indicate that the variation in specific heat ratio had no significant effect on the mass flow ratio over the range of various vortex flow conditions and radius ratios investigated. The mass flow ratio is defined as the ratio of mass flow rate as computed by a two-dimensional analysis to that computed by a one-dimensional analysis. The critical velocity ratio had only a small effect on mass flow ratio. Very little or no difference was found in computed mass flow between a one- and two-dimensional analysis for all free-vortex cases. For non-free-vortex cases, the mass flow ratio decreased as swirl velocity distribution deviated from the free-vortex condition and as radius ratio decreased. This decrease in mass flow ratio became more pronounced as flow angle increased.

INTRODUCTION

In the annulus of a turbomachine, there is some variation in the mass flow per unit area from the hub to the tip of a blade row. This is due to the specified radial variation in tangential velocity and the balance of forces that must exist in the flow. Although this radial variation in mass flow exists, an arithmetic-mean-section velocity diagram is often used to compute the entire mass flow. Such a one-dimensional assumption is reasonable with relatively high hub-to-tip radius ratio turbines (about 0.85 or greater), where the radial variation in flow is small. For lower hub-to-tip radius ratio turbines, however, the radial variation in flow is substantial, and the mean-section flow conditions

may not represent true average conditions for computing the mass flow.

In the past, turbines were usually designed for free vortex conditions (rV_u = constant). This type of design results in radially constant axial velocity and nearly constant mass flow per unit area. Recently, however, there is increased interest in non-free-vortex designs. In axial turbines with low hub-to-tip radius ratio and highly loaded blades, it may be desirable to use non-free-vortex designs since, for certain conditions, they can result in improved aerodynamic conditions for the rotor hub section. Reference 1 shows that non-free-vortex designs have been used to obtain improved turbine performance. The non-free-vortex designs can have large radial variation in mass flow rate per unit area due to a large nonlinear radial gradient in axial velocity. Thus, the mean section flow conditions will not represent the true average conditions for such cases. Considerable error in computed mass flow may occur if such a turbine is designed on the basis of one-dimensional mean-section flow conditions.

This report examines the difference in computed mass flow between a one-dimensional analysis and a two-dimensional one to see to what extent a one-dimensional analysis is still valid. A number of swirl velocity distributions (i.e., radial variation of V_u) will be specified and examined over a range of radius ratios for several values of specific heat ratio, flow angle, and critical velocity ratio. The results of this study can be used to indicate the error in computed mass flow associated with turbine mean-section analyses, which are often used for preliminary design studies, and to provide compensating correction factors.

METHOD OF ANALYSIS

The radial variation of the tangential component of velocity for most axial flow turbines is usually specified as $V_u = Kr^N$. All symbols are defined in appendix A. When the swirl distribution exponent N is set equal to -1, the flow condition is normally referred to as free vortex. Free-vortex designs have been used so often that all other exponents are usually referred to as non-free-vortex designs. In this report, a range of exponents from -2 to +1, including the constant-flow-angle case, is investigated in order to determine the ratio of mass flow as computed by a two-dimensional analysis to that computed by a one-dimensional analysis.

For a one-dimensional analysis at the arithmetic-mean section of a blade row, the mass flow is given as

$$w_m = \rho_m V_{xm}\left(r_t^2 - r_h^2\right) \tag{1}$$

The mass flow for the two-dimensional case may be written as

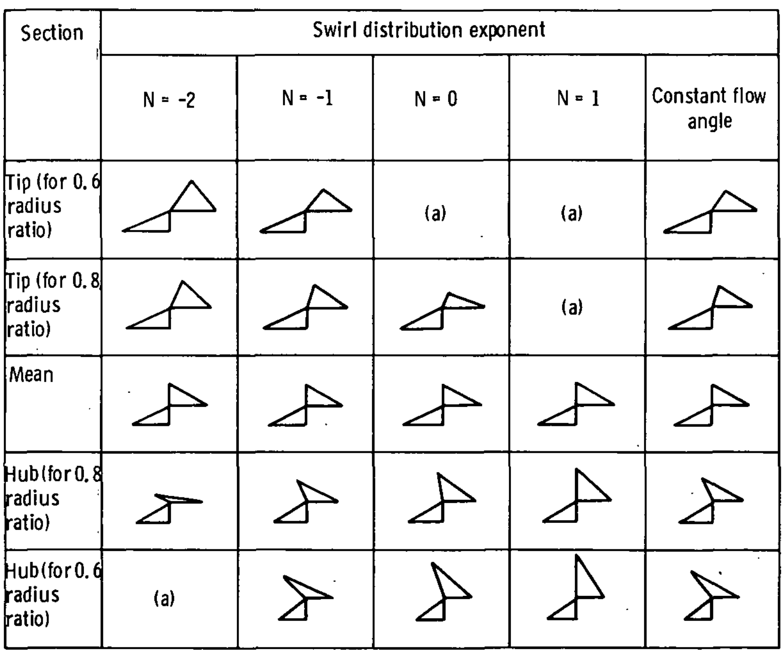
$$w = \int_{r_h}^{r_t} \rho V_x 2\pi r \, dr \tag{2}$$

The ratio of equations (2) to (1) yields the desired mass flow ratio

$$\frac{W}{W_m} = \frac{2}{1 - \left(\frac{r_h}{r_t}\right)^2} \int_{r_h/r_t}^{1} \left(\frac{\rho}{\rho_m}\right)\left(\frac{V_x}{V_{xm}}\right)\left(\frac{r}{r_t}\right) d\left(\frac{r}{r_t}\right) \tag{3}$$

The equations for the axial velocity ratio V_x/V_{xm} were obtained from reference 2. The evaluation of all the terms in the integral for various flow conditions is given in appendix B.

From the equations of appendix B, the radial variations in velocity for the various swirl velocity distributions can be determined. The radial variations in velocity diagrams associated with several of the swirl distributions being studied herein are illustrated in figure 1. For simplicity, the rotor exit flow was assumed to have no swirl velocity component, and the radial variations in velocity appear only at the stator exit (rotor inlet). This figure shows the velocity diagrams for a blade mean section, which



[a]No real value for axial velocity.

Figure 1. - Radial variation of velocity diagrams for various swirl distributions. Stator mean-section exit angle, 60°.

is the same for all cases, and for the hub and tip sections of blades with radius ratios of 0.8 and 0.6.

The free-vortex ($N = -1$) case is seen to have constant axial velocity from hub to tip. The constant-flow-angle case shows a small decrease in axial velocity with increasing radius from hub to tip. The other cases, $N = -2$, 0, and 1, show large changes in axial velocity as a function of radius. For $N < -1$, the axial velocity increases with increasing radius from hub to tip, while for $N > -1$, the axial velocity decreases with increasing radius. Where diagrams are not shown, the axial velocity has become imaginary, and there is no real solution for that point. Limitations associated with this situation are discussed as part of the analysis results.

A number of independent variables were used in computing the mass flow ratio for several values of N (-2, -1, 0, and 1) and for constant flow angle. The independent variables are specific heat ratio γ, critical velocity ratio V_m/V_{cr}, flow angle α_m, and the radius ratio r_h/r_t. These were varied over the following ranges:

γ: 1.3, 1.4, and 1.67

$\frac{V_m}{V_{cr}}$: 0.4, 0.6, 0.8, and 1.0

α_m: 45^o, 55^o, 65^o, and 75^o

$\frac{r_h}{r_t}$: 0.5 to 1.0

RESULTS OF ANALYSIS

An analysis of the mass flow ratio for a turbine annulus was made using a wide range of independent variables. The results of the analysis will be presented as follows: The effect of specific heat ratio on the mass flow ratio will be discussed first. This will be followed by a discussion of the limits of real solutions to the mass flow ratio equation for some of the swirl distribution cases investigated. The effect of radius ratio on mass flow ratio will then be presented and discussed for each swirl distribution case over the ranges of flow angle and velocity ratio considered. Finally, the results are extended to all swirl distribution exponents in the range considered by crossplotting mass flow ratio against swirl distribution exponent for two values of radius ratio and all angles.

Specific Heat Ratio

The effect of specific heat ratio γ on the mass flow ratio was examined. Three specific heat ratios were considered for this analysis: 1.3, 1.4, and 1.67. The effect of these ratios on the mass flow ratio was insignificant for all swirl distribution cases considered (usually less than 0.3 percent). Therefore, the figures in this report will be presented for the specific heat ratio of 1.4 only.

Radius Ratio Limit

Figure 2 shows the variation of radius ratio limit with swirl distribution exponent for various flow angles. There are no limits for the exponent $N = -1$ and the constant-flow-angle case. The radius ratio limit is that radius ratio below which no real solution exists for the axial velocity ratio V_x/V_{xm}, as determined from equations (B10) or (B12). Below the limiting value, therefore, the mass flow ratio (eq. (3)) cannot be evaluated. The significance of this limit is that it is not possible to have the indicated swirl velocity distribution for blades with radius ratios less than the limiting value. As seen from figure 2, the limiting value of radius ratio decreases for all angles as the swirl distribution exponent approaches the free-vortex condition, $N = -1$. There is also a decrease in the radius ratio limit with a decrease in flow angle. For lower radius ratios and large flow angles there is little opportunity for selecting a turbine design with a swirl velocity distribution much different from the free-vortex or constant-flow-angle design cases.

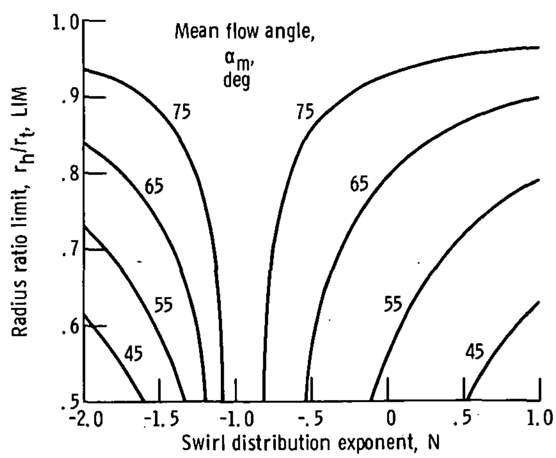
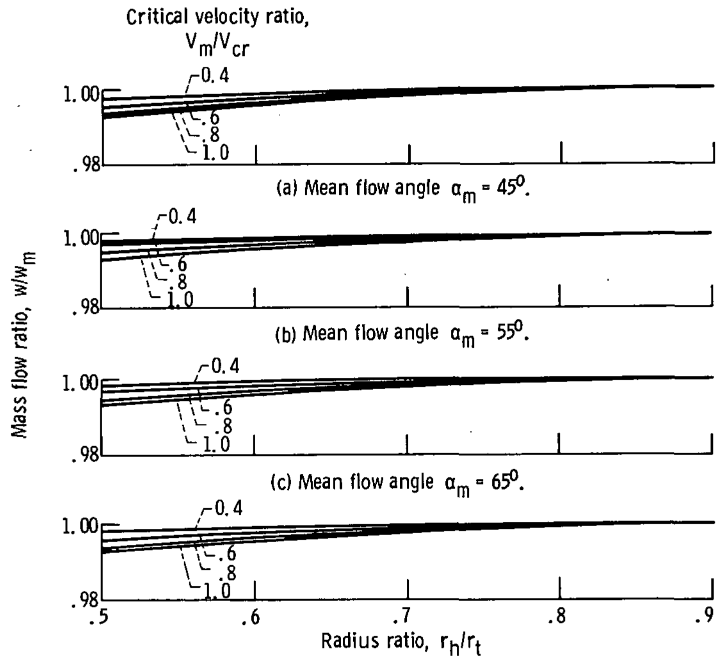


Figure 2. - Variation of radius ratio limit with swirl distribution exponent.

Free-Vortex Flow, N = -1

Using this exponent in equation (B10) results in a radially constant axial velocity. Figure 3 shows the mass flow ratio as a function of radius ratio for flow angles of 45°, 55°, 65°, and 75° and for various values of critical velocity ratio.

The mass flow ratio decreases with decreasing radius ratio, but the change is small. At a radius ratio of 0.50 and critical velocity ratio of 1.0, the mass flow ratio is 0.993 for all angles investigated. It can be seen that the difference in mass flow rate is negligible from one flow angle to another for each velocity ratio and from one velocity ratio to another for each angle. Thus, there is very little difference between a one- and two-dimensional approach for the free-vortex condition in the range of radius ratios covered in this analysis.



(a) Mean flow angle $\alpha_m = 45^\circ$.

(b) Mean flow angle $\alpha_m = 55^\circ$.

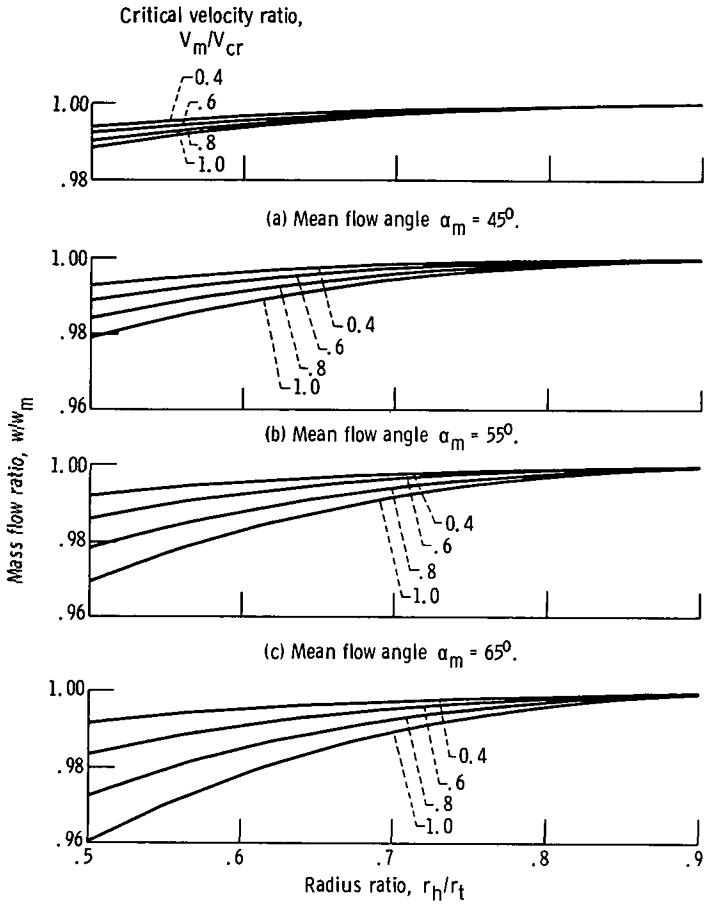
(c) Mean flow angle $\alpha_m = 65^\circ$.

(d) Mean flow angle $\alpha_m = 75^\circ$.

Figure 3. - Variation of mass flow ratio with radius ratio. Swirl distribution exponent N = -1.

Non-Free Vortex, α = Constant

Figure 4 shows the variation of mass flow ratio with radius ratio for a number of flow angles and critical velocity ratios. The decrease in mass flow ratio with a decrease in radius ratio, for all values of critical velocity ratio, is somewhat larger than for



(d) Mean flow angle α_m = 75°.

Figure 4. - Variation of mass flow ratio with radius ratio. Constant flow angle.

free-vortex flow. For a flow angle of 55° and a velocity ratio of 1.0, the mass flow ratio is 0.979 at a radius ratio of 0.50. This compares with 0.993 at the same condition for free-vortex flow. There is also a decrease in mass flow ratio with an increase in flow angle. At a radius ratio of 0.50 and a velocity ratio of 1.0, the mass flow ratio was about 3 percent lower for a flow angle of 75° than for a flow angle of 45°. Variations in the critical velocity ratio have little effect on the mass flow ratio at any radius ratio. The greatest difference occurs at a flow angle of 75° and a radius ratio of 0.5, where there is about a 1 percent difference in mass flow ratio between each value of the critical velocity ratio used.

Non-Free Vortex, N = 0

Figure 5 shows the variation of mass flow ratio with radius ratio for a number of flow angles and critical velocity ratios. The decrease in mass flow ratio with a decrease in radius ratio for any velocity ratio is much larger than the two previous cases. The figure also shows a significant decrease in mass flow ratio with an increase in flow angle. Note too, there are no solutions below radius ratios of about 0.56, 0.79, and 0.93 for flow angles of 55°, 65°, and 75°, respectively. There is very little change in the mass flow ratio for variations in the critical velocity ratio at any given radius ratio.

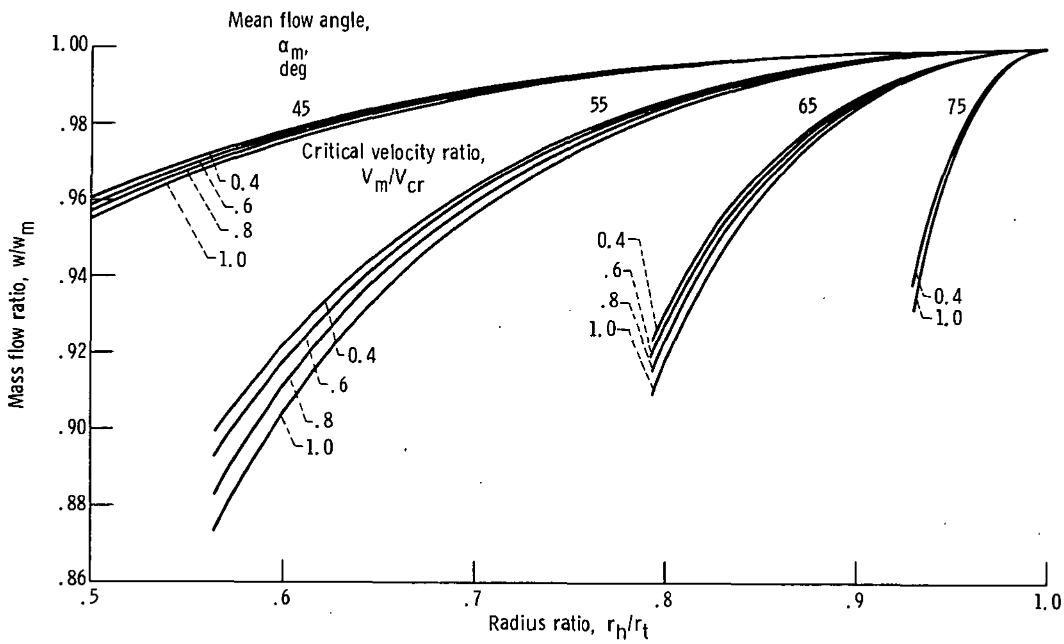


Figure 5. - Variation of mass flow ratio with radius ratio. Swirl distribution exponent N = 0.

Non-Free Vortex, N = 1

Figure 6 shows the variation of mass flow ratio with radius ratio for various flow angles and critical velocity ratios. Only the 0.4 and the 1.0 critical velocity ratio curves are plotted since the difference between the velocity ratios investigated for any flow angle is small. The decrease in mass flow ratio with radius ratio is the largest for all cases considered. The decrease in mass flow ratio with an increase in flow angle is also the largest for this case. There are no solutions to the equations below radius ratios of about 0.63 for 45°, 0.79 for 55°, 0.90 for 65°, and 0.95 for 75°.

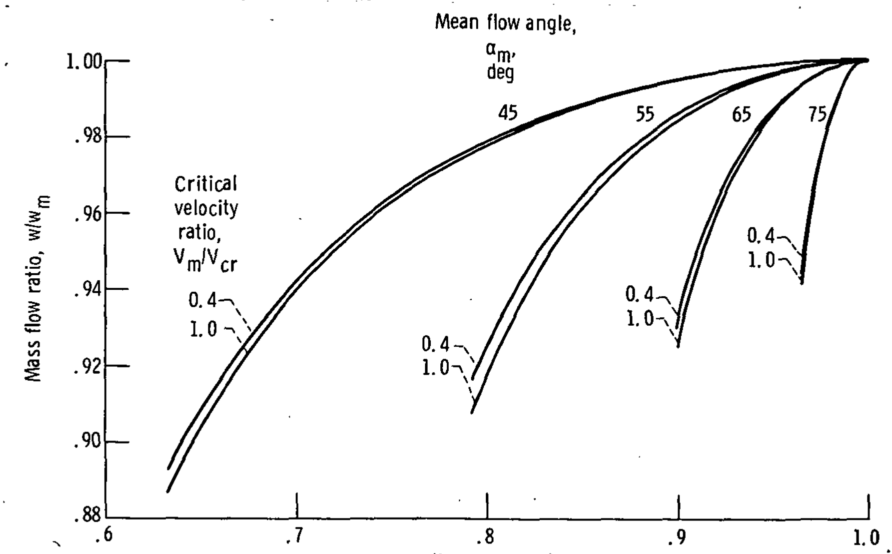


Figure 6. - Variation of mass flow ratio with radius ratio. Swirl distribution exponent N = 1.

Non-Free Vortex, N = -2

Figure 7 shows the mass flow ratio as a function of the radius ratio for various flow angles and critical velocity ratios. As in figure 6, only the 0.4 and 1.0 critical velocity ratio curves are plotted since the difference between the velocity ratios investigated for any flow angle is small. In this case, the effect of critical velocity ratio is reversed (i.e., the 0.4 velocity ratio produces the lowest mass flow ratio). The figure shows a decrease in mass flow ratio with a decrease in radius ratio. However, it is not as severe as in the previous figure where N = 1. With an increase in the flow angle, there

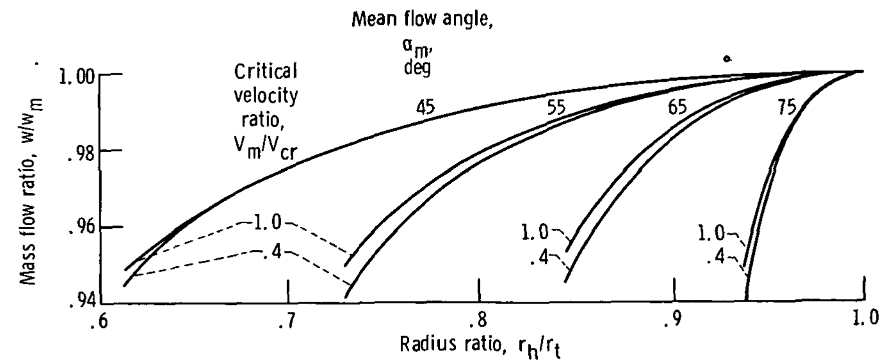


Figure 7. - Variation of mass flow ratio with radius ratio. Swirl distribution exponent N = -2.

is a large drop in the mass flow ratio. There are no solutions to the equations below radius ratios of about 0.61 for 45°, 0.73 for 55°, 0.84 for 65°, and 0.94 for 75°.

Effect of Swirl Distribution Exponent

In order to show the results of the analysis for all swirl velocity exponents in the range studied, cross plots of mass flow ratio versus exponent were made. These are shown in figure 8 for radius ratios of 0.8 and 0.6. Since it has been shown that critical velocity ratio has only a small effect on mass flow ratio, the critical velocity ratio was held constant at a value of 1.0 for this figure. In figure 8(a) with a radius ratio of 0.8, the curves for all flow angles have the same maximum value of 0.999 for the mass flow ratio at the swirl distribution exponent N = -1 (free vortex). In figure 8(b) with a radius

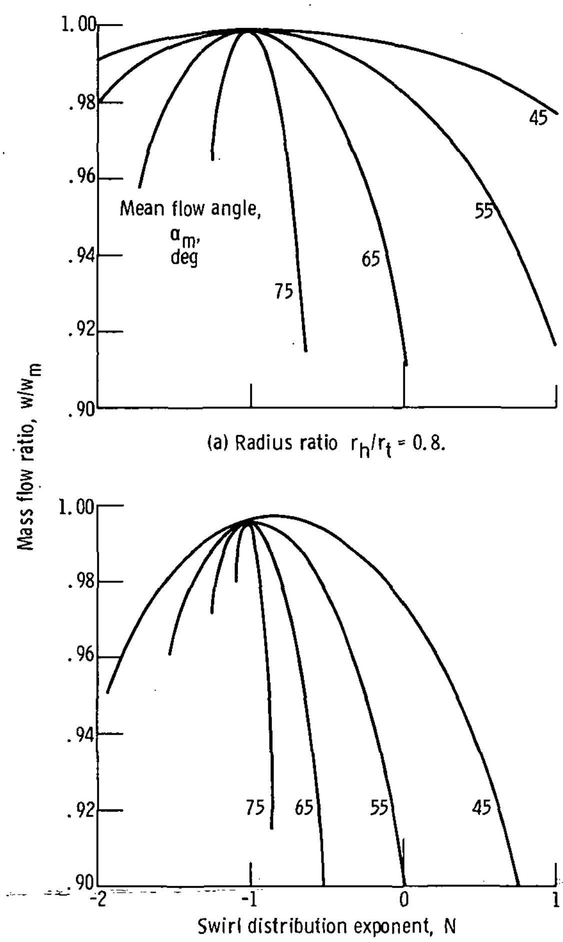


Figure 8. - Variation of mass flow ratio with swirl velocity exponent.

ratio of 0.6, the curves for all flow angles have the same value of 0.996 for the mass flow ratio at the exponent $N = -1$. This is the maximum value for all flow angles shown except for 45°, where the maximum value of mass flow ratio is slightly higher and is displaced to $N = -0.8$. For both sets of curves, the decrease in mass flow ratio from its maximum value becomes more rapid as flow angle increases.

It is seen from figure 8, as well as from previous figures, that the mass flow ratio is nearly 1 for a free-vortex ($N = -1$) design under all conditions of interest. Thus, a one-dimensional computation of mass flow rate for a free-vortex design is always quite accurate. For non-free-vortex designs, in general, the accuracy of the one-dimensional computation becomes less as the swirl velocity distribution increasingly deviates from the free-vortex condition. In addition, there is a further decrease in accuracy as the flow angle becomes larger and the radius ratio becomes smaller. For many non-free-vortex cases, the inaccuracy of a one-dimensional computation of mass flow rate could be on the order of 5 to 10 percent or more. In the special case of a constant flow angle, the inaccuracy of a one-dimensional computation would be less than 5 percent even for large angles and low radius ratios. Note that, in all cases, the mass flow ratio is less than 1; therefore, the mass flow in a turbine will always be less than that computed by a one-dimensional mean-section analysis.

SUMMARY OF RESULTS

The difference in mass flow between a one-dimensional and a two-dimensional analysis using various radial distributions of swirl velocity was investigated. The independent variables used in the analysis were swirl velocity distribution, flow angle, critical velocity ratio, radius ratio, and specific heat ratio. The results of the analysis can be summarized as follows:

1. The specific heat ratio had no significant effect on the mass flow ratio for the radius ratios, flow angles, and swirl velocity distributions investigated.

2. The mass flow ratio is relatively insensitive to changes in critical velocity ratio at any given radius ratio for all flow angles and swirl velocity distributions investigated.

3. There is little or no difference between a one- and two-dimensional approach in determining mass flow for the free-vortex case under all conditions investigated.

4. For the non-free-vortex cases, the mass flow ratio decreases significantly as the swirl velocity exponent increasingly deviates from the free-vortex value of $N = -1$. The rate of decrease becomes greater as flow angle increases.

5. For the non-free-vortex cases, the mass flow ratio decreases as radius ratio decreases. The rate of decrease becomes greater as flow angle increases.

6. Except for the constant-flow-angle case, all non-free-vortex flow cases had radius ratio limits below which no real solution exists for the mass flow ratio. These radius ratio limits increase with increasing flow angle.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, August 22, 1972,
501-24.

APPENDIX A

SYMBOLS

g conversion constant, 1; 32.174 lbm-ft/lbf-sec^2

h specific enthalpy, J/kg; Btu/lb

J conversion constant, 1; 778.029 ft-lb/Btu

K proportionality constant

N swirl distribution constant

P pressure, N/m^2; lb/ft^2

r radius, m; ft

s specific entropy, J/(kg)(K); Btu/(lb)($^{\circ}$R)

T temperature, K; $^{\circ}$R

V absolute velocity, m/sec; ft/sec

v specific volume, m^3/kg; ft^3/lb

w mass flow rate, kg/sec; lb/sec

α absolute gas flow angle, angle between absolute velocity vector and meridional plane

γ ratio of specific heat at constant pressure to specific heat at constant volume

ρ gas density, kg/m^3; lb/ft^3

Subscripts:

cr conditions corresponding to Mach 1

h hub

LIM limit

m mean

me meridional

t tip

u tangential component

x axial component

Superscript:

' absolute total state

APPENDIX B

ANALYTICAL PROCEDURE

The equations for the axial velocity ratio V_x/V_{xm} used in the calculations of mass flow ratio were obtained from reference 2. However, they are repeated here as well as their development for the convenience of the reader.

The radial equilibrium equation for a turbine blade row is

$$\frac{g}{\rho}\frac{dP}{dr} = \frac{V_u^2}{r} - \frac{V_{me}^2}{r_{me}} \cos \alpha_{me} - \frac{dV_{me}}{dt} \sin \alpha_{me} \tag{B1}$$

Since the meridional streamline curvatures $1/r_{me}$ and inclination angles α_{me} are both quite small for axial or near axial turbines, we may neglect the last two terms. We can then write

$$\frac{g}{\rho}\frac{dP}{dr} = \frac{V_u^2}{r} \tag{B2}$$

This is known as the "simple" radial equilibrium equation. Now since we can assume the meridional streamline slope to be zero so that no radial component of velocity exists, then the total enthalpy can be written as

$$h' = h + \frac{V_u^2}{2gJ} + \frac{V_x^2}{2gJ} \tag{B3}$$

Differentiating with respect to radius and using $dh = T\,ds + \frac{1}{J} v\,dP$ and $\rho = 1/v$ we have

$$\frac{dh'}{dr} = T\frac{ds}{dr} + \frac{1}{J\rho}\frac{dP}{dr} + \frac{1}{2gJ}\frac{dV_u^2}{dr} + \frac{1}{2gJ}\frac{dV_x^2}{dr} \tag{B4}$$

Using the "simple" radial equilibrium expression (B2) and with the additional assumption that total enthalpy and the entropy are radially constant we get

$$\frac{V_u^2}{r} + \frac{1}{2}\frac{d\left(V_u^2\right)}{dr} + \frac{1}{2}\frac{d\left(V_x^2\right)}{dr} = 0 \tag{B5}$$

At this point, it is necessary to specify a relation between V_u and V_x or between V_u or V_x and r. The relation most frequently used is that of swirl velocity with radius and is stated as

$$V_u = Kr^N \tag{B6}$$

Substitution of equation (B6) and its differential with respect to radius into equation (B5) and multiplying through by dr gives

$$K^2 r^{2N-1}\,dr + NK^2 r^{2N-1}\,dr + \frac{1}{2}\,d\left(V_x^2\right) = 0 \tag{B7}$$

Integration of equation (B7) between the limits of V_x to V_{xm} and r to r_m yields

$$V_x^2 - V_{xm}^2 = \left(\frac{N+1}{N}\right) K^2\left(r_m^{2N} - r^{2N}\right) \tag{B8}$$

Dividing through by V_{xm}^2 and substituting the following relations into equation (B8):

$$K^2 = \frac{V_u^2}{r^{2N}} = \frac{V_{um}^2}{r_m^{2N}} \qquad \text{(from eq. (B5))}$$

and

$$\tan \alpha_m = \frac{V_{um}}{V_{xm}}$$

give

$$\frac{V_x^2}{V_{xm}^2} = 1 - \left(\frac{N+1}{N}\right)\tan^2\alpha_m\left[\left(\frac{r}{r_m}\right)^{2N} - 1\right] \tag{B9}$$

Using the radius relations

$$\frac{r}{r_m} = \frac{r}{r_t}\,\frac{r_t}{r_m} \quad \text{and} \quad r_m = \frac{r_h + r_t}{2}$$

we finally obtain

$$\frac{V_x}{V_{xm}} = \left\{1 - \left(\frac{N+1}{N}\right)\tan^2\alpha_m\left[\left(\frac{r}{r_t}\right)^{2N}\left(\frac{2}{\dfrac{r_h}{r_t} + 1}\right)^{2N} - 1\right]\right\}^{1/2} \tag{B10}$$

Equation (B10) is not valid for the special case of $N = 0$ ($V_u = K$ from eq. (B6)). For this case equation (B5) becomes

$$\frac{K^2}{r} + \frac{1}{2}\frac{d\left(V_x^2\right)}{dr} = 0 \tag{B11}$$

Integrating this equation with the same limits as before and using the same radius relations as in equation (B10) yield

$$\frac{V_x}{V_{xm}} = \left\{1 - 2 \tan \alpha_m \ln\left[\left(\frac{r}{r_t}\right)\left(\frac{2}{\frac{r_h}{r_t} + 1}\right)\right]\right\}^{1/2} \tag{B12}$$

For a radially constant flow angle, we can write $V_u = V_x \tan \alpha$, and substituting this expression in equation (B5) gives

$$\frac{V_x^2 \tan^2 \alpha}{r} + \frac{1}{2} \tan^2 \alpha \frac{d\left(V_x^2\right)}{dr} + \frac{1}{2}\frac{d\left(V_x^2\right)}{dr} = 0 \tag{B13}$$

Integration of this equation with its proper limits gives

$$\frac{V_x}{V_{xm}} = \left(\frac{r}{r_m}\right)^{-\sin^2 \alpha} \tag{B14}$$

The equation for the static density ratio is developed as follows:

$$\frac{\rho}{\rho_m} = \frac{\rho/\rho'}{\rho_m/\rho'} \tag{B15}$$

and

$$\frac{\rho}{\rho'} = \left[1 - \frac{\gamma - 1}{\gamma + 1}\left(\frac{V}{V_{cr}}\right)^2\right]^{1/(\gamma-1)} \tag{B16}$$

Thus,

$$\frac{\rho}{\rho_m} = \frac{\left[1 - \frac{\gamma - 1}{\gamma + 1}\left(\frac{V}{V_{cr}}\right)^2\right]^{1/(\gamma-1)}}{\left[1 - \frac{\gamma - 1}{\gamma + 1}\left(\frac{V_m}{V_{cr}}\right)^2\right]^{1/(\gamma-1)}} \tag{B17}$$

Now,

$$\left(\frac{V}{V_{cr}}\right)^2 = \left(\frac{V}{V_m}\right)^2\left(\frac{V_m}{V_{cr}}\right)^2 = \left(\frac{V_m}{V_{cr}}\right)^2\left[\left(\frac{V_x}{V_m}\right)^2 + \left(\frac{V_u}{V_m}\right)^2\right] \tag{B18}$$

Also,

$$V_m^2 = \frac{V_{xm}^2}{\cos^2\alpha_m}$$

and

$$V_m^2 = \frac{V_{um}^2}{\sin^2\alpha_m} \tag{B20}$$

Substituting equations (B18) to (B20) into equation (B17) gives

$$\frac{\rho}{\rho_m} = \left\{\frac{1 - \frac{\gamma - 1}{\gamma + 1}\left(\frac{V_m}{V_{cr}}\right)^2\left[\left(\frac{V_x}{V_{xm}}\right)^2\cos^2\alpha_m + \left(\frac{V_u}{V_{um}}\right)^2\sin^2\alpha_m\right]}{1 - \frac{\gamma - 1}{\gamma + 1}\left(\frac{V_m}{V_{cr}}\right)^2}\right\}^{1/(\gamma-1)} \tag{B21}$$

where

$$\frac{V_u}{V_{um}} = \left(\frac{r}{r_m}\right)^N = \left(\frac{r}{r_t}\right)^N\left(\frac{2}{\frac{r_h}{r_t} + 1}\right)^N$$

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